Strategies for space rendezvous on Lunar Distant Retrograde Orbits

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Abstract

This research is motivated by a possible deployment of an inhabited space station on a cislunar orbit. This paper describes space rendezvous on Lunar Distant Retrograde Orbits (DRO). More precisely, the core study of this research tackles the construction of a DRO, and the rendezvous itself. Then further work has been conducted including safety analysis during the rendezvous operations and optimization of DRO computation.

1 Context

In order to understand what DRO are, one must start with the Lagrangian points of the Earth-Moon system. They are equilibrium points of the gravitational field in the Circular Restricted Three-Body Problem (CR3BP) whose dynamics is detailed by Koon, Lo, Marsden and Ross[1]. The three bodies at stake here are the Earth, the Moon, and a particle. There are five Lagrangian points : three on the x-axis of the Earth-Moon synodic reference frame, called collinear points, and two forming an equilateral triangle with the Earth and the Moon, called triangular or equilateral points.

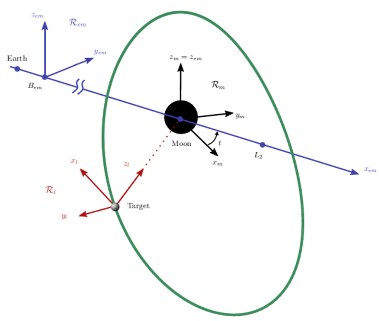

Figure 1: Reference frames in cislunar space. In blue: the Earth-Moon synodic reference frame. In black: the Lunar-Centered Inertial frame (LCI). In red: the NRO Local Vertical, Local Horizontal (LVLH) frame for rendezvous operations. In green: the NRO of the target, as seen in MCI. (from [2])

The synodic frame is a rotating frame with origin at the center of mass of the Earth-Moon such that the Earth and the Moon are fixed points on the x-axis. The z-axis is orthogonal to the plane of motion of these bodies and the y-axis is obtained by the right-hand rule, as showed in Figure 1. Around the collinear points, there exist several families of orbits and solutions of the CR3BP (as showed in Figure 2) among which can be found HALO and Lyapunov orbits, that are commonly studied; but also Near Rectilinear Orbits (NRO) and DRO, whose study is state-of-the-art because of the NASA interest.

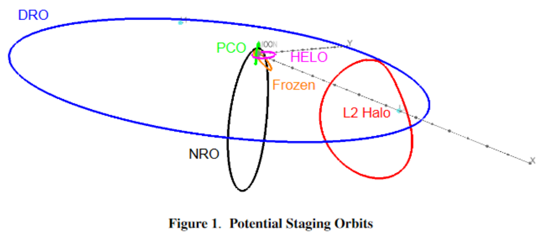

Figure 2: Orbits Comparison (from [3])

DRO are planar solutions of the CR3BP which traverse the Moon in a clockwise way in the Earth-Moon rotating frame. Due to their long-term stability, DRO are seen as possible destinations for captured asteroids in the Asteroid Redirect Mission (ARM). Nowadays studies tend to target DRO and NRO as possible destination for the Deep Space Gateway (DSG) as a successor or amplitude of the International Space Station (ISS) because of their long-term stability and because the Lagrangian points (in particular L2) can be seen as logistic hubs for human low fuel consumption missions in interplanetary space.

A rendezvous is defined as the sequence of maneuvers that a chaser vehicle performs in order to bring itself along a target vehicle, which is passive and non-maneuvering. The resulting motion of the chaser seen from a reference frame that is centered on the target is defined as relative motion. All the rendezvous with the ISS performed by NASA and ESA are based upon the assumption of two vehicles operating on a near circular orbit on a strong gravitational field due to a massive central body. However neither of these conditions are present in a cislunar environment. Thus, the constraints and safety procedures derived by operating on a central gravity field are no longer in effect.

2 Problem Statement

Since International Space Exploration Coordination Group (ISECG) is willing to pursue space exploration [4], a space base is valuable in order to launch interplanetary missions at a lower cost than the one launched from Earth. Yet a space station must be re-supplied regularly and thus rendezvous on DRO need to be explored.

The first step to complete is the construction of some DRO, because there are no data array gathering their characteristics. With some known DRO the others can be extrapolated.The second step is to adapt and compare classical methods for rendezvous (Clohessy-Wiltshire, Linearized Relative Targeting and Straight-Line Targeting) to DRO, and if need be invent another method. Further steps are optimization of the rendezvous strategy, taking into account safety analysis and addressing a compromise between the time of flight, the position of the rendezvous and the required ΔV.

3 First results

3.1 Construction of a DRO

A classical method to construct an orbit is to choose an initial state vector and use differential correction to obtain a whole orbit (chapter 4 of [1]). Since DRO are periodic, planar and symmetric about x-axis orbits, the initial state vector has been chosen at the intersection of the orbit and the x-axis. Therefore, among of the six components of the state vector :

$$X_0 = [x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]^T \quad (1)$$

only x_0 and $\dot{y}$ are not nulls.

The initial state that has been implemented in this iterative process is the one used by Murakami and Yamanaka [5] as it follows :

$$X_0 = [x_m - a_x, 0, 0, 0, \dot{y}_0, 0]^T \ , \ x_m = 1 - \mu \quad (2)$$

where x_m and $a_x = A_x/L$ are the position of the Moon and the dimensionless amplitude of the DRO in the Earth-Moon synodic frame (L=384,400 km and is the mean radius of the Moon orbit). Yet there is no insight about $\dot{y}_0$. That is why Jacobi's constant (first integral of equation of the CR3BP) has been used. In fact, Jacobi's constant is the only known conserved quantity in the CR3BP; thus it has a specific value for any DRO :

$$C_j = x^2 + y^2 + 2(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}) - (\dot{x}^2 + \dot{y}^2) \quad (3)$$

Using the discussed initial state vector in Equation1:

$$C_j(t=0) = x_0^2 + 2(\frac{1-\mu}{\mid x_0 + \mu \mid} + \frac{\mu}{\mid 1 - \mu - x_0 \mid}) - \dot{y}_0^2 \quad (4)$$

$$\mu = \frac{m_2}{m_2 + m_1} \quad (5)$$

m_1 and m_2 are the masses of the Earth and the Moon
C_j : Jacobi's constant
x_0 : initial position in the synodic framework
$\dot{y}_0$: vertical component of the initial velocity in the synodic frame

Yet there is no means to determine the value of Jacobi's constant corresponding to a DRO as expressed in Equation4. However, for different DRO, the values of several Jacobi's constants and their associated initial positions x_0 have been calculated by Michel Hénon [6]. With these data, Hénon's DRO have been computed with Matlab (Figure 3).

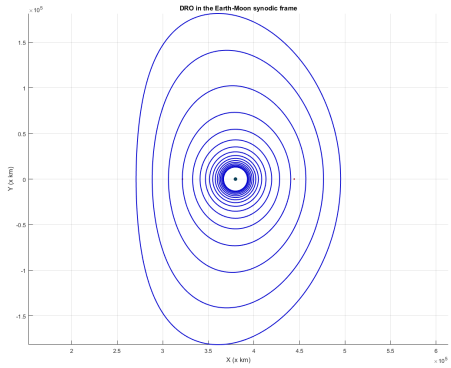

Figure 3: Results of a numerical simulation of DRO using differential correction with Matlab

In order to describe a rendezvous on them, it is necessary to compute any DRO. So with a continuation algorithm other DRO with various sizes have been computed, with amplitude A_x ranging from $16 \cdot 10^3$ km to $350 \cdot 10^3$ km. The results are shown in Figure 4.

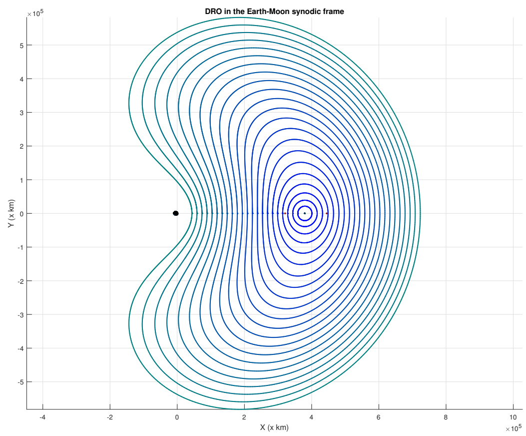

Figure 4: DRO in the Earth-Moon synodic frame, computed with Matlab

3.2 Further construction of DRO

The continuation algorithm starts with the two smallest known DRO. Then it provides a third orbit by repeating the difference in amplitude between the first two orbits. Thus, a whole family of orbits can be generated, knowing only two orbits of that family. Hénon's DRO are then included in the continuation algorithm as landmarks. The continuation has also been proceeded in reverse direction to get smaller DRO.

Using Hénon's numerical values, it is possible to compare the Jacobi's constants of computed orbits with analytical values obtained with a Taylor expansion of C_j in the vicinity of the Moon. For this research, a 6^{th} order Taylor expansion of C_j with respect to $\mu^{1/3}$ has been calculated by using the new set of variables $x = 1 - \mu + \mu^{1/3} \cdot \xi$ and $y = \mu^{1/3} \cdot \eta$. One can show that:

$$C_j = 3 + \Gamma\mu^{1/3} + k_3\mu + k_4\mu^{4/3} + k_5\mu^{5/3} + k_6\mu^2 + \mathrm{o}(\mu^2) \quad (6)$$

Where value of Γ are given in Hénon's table.
$k_3 = -4 - 2\xi^3 + 3\xi\eta^2$
$k_4 = 2\xi^4 - 6\xi^2\eta^2 + 2\xi + 3/4\eta^4$
$k_5 = -2\xi^5 + 10\xi^3\eta^2 - 2\xi^2 - 15/4\xi\eta^4 + \eta^2$
$k_6 = 1 + 2\xi^6 - 15\xi^4\eta^2 + 2\xi^3 + 45/4\xi^2\eta^4 - 3\xi\eta^2 - 15/8\eta^6$

This expression is used to check whether or not computed orbits are DRO. However it is only valid for orbits with low amplitude. Periods of computed orbits

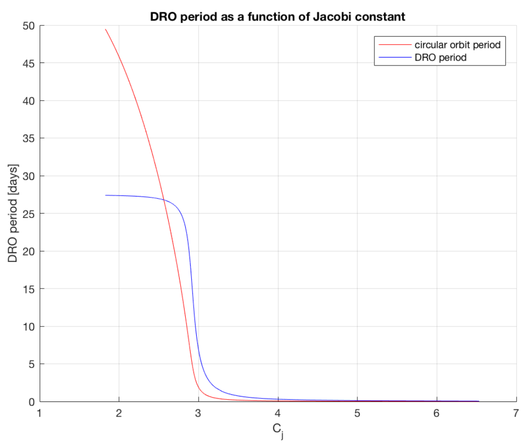

Figure 5: Period of computed DRO (blue) and a Moon-centered circular orbit (red) as functions of their associated Jacobi's constant

are gathered in Figure 5.As shown in this curve, orbits with high C_j (thus low a_x) have the same periods as those of Moon-centered circular orbits. This curve is also identical to that of DRO from [7]. This comparison ensures that computed orbits are DRO. A broad computation of DRO has been done to estimate the value of C_j (thus the initial velocity) for various amplitudes (Figure 6). The relevant equations are recalled here :

$$X_0 = [x_o, 0, 0, 0, \dot{y}_0, 0]^T \text{ with } x_0 = 1 - \mu - a_x \quad (7)$$

$$\dot{y}_0 = \sqrt{x_0^2 + 2\left(\frac{1-\mu}{\mid x_0 + \mu \mid} + \frac{\mu}{\mid x_0 - 1 + \mu \mid}\right) - C_j(t=0)} \quad (8)$$

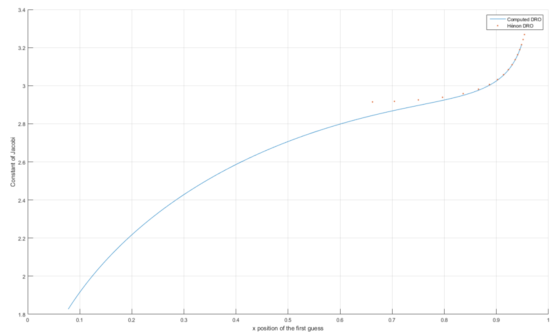

Figure 6: Jacobi's constant C_j as a function of x_0. Reference values from Hénon's table in red

A fitting curve was obtained using Matlab polynomial interpolation. Its validity domain begins with Ax = 15x10^3 km and ends around 350x10^3 km. Over this region, the mean squared error is 3.7x10^{-7} .

$$C_j(x_0) = b_n x_0^n + ... + b_1 x_0 + b_0 \ .$$

n	Coefficient (b_n)
10	2.2134845x10^3
9	-1.0492761x10^4
8	2.15062444x10^4
7	-2.49398094x10^4
6	1.79776019x10^4
5	-8.32412566x10^3
4	2.461494428x10^3
3	-4.397402x10^2
2	3.69677x10^1
1	2.9419
0	1.5123

3.3 A Rendezvous on a DRO

A space rendezvous is a sequence of orbital maneuvers during which a chaser comes close to a target and aims at a docking, that is to say a soft contact between the two of them. A special reference frame is designed for space rendezvous : the Local Vertical Local Horizontal (LVLH). It is centered on the target so as to consider only the relative motion of the chaser. The z-axis is directed toward the center of the main primary (the Moon has been chosen), and is named the Altitude. The y-axis is chosen to be opposite to the cross-product of the z-axis and the target's velocity, it is called the Cross Track. The x-axis is the cross-product of the two other, in order to obtain a direct orthonormal coordinate frame, it is called Crossbar or Downrange.(see Figure 1)

Kuljit Mand [8] describes three algorithms to compute a rendezvous, but none are specially designed for DRO. They are simplifications of the relative equations of motion. They are mainly used to determine the ΔV that the chaser must provide at each step of the rendezvous. Yet, in order to get the ΔV, the relative equations of motion must be linearized to the form of $\dot{X} = AX$ where X is the relative state and A is the system's dynamical matrix, which needs to be independent from time.

The Clohessy-Wiltshire (CW) approach uses the circular hypothesis and thus is simple to compute, but is only efficient and accurate in the vicinity of one of the primaries.

$$\dot{X} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2n \\ 0 & -n^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3n^2 & -2n & 0 & 0 \end{bmatrix} X \tag{9}$$

with

$$n = \frac{2\pi}{T} = \sqrt{\frac{G \cdot M_{CentralBody}}{a^3}} \tag{10}$$

The Straight-Line (SL) algorithm it is simple to implement since gravitational effects are neglected. Therefore is used when the target and the chaser are close enough and far enough from the primaries. Mathematically it is close to the CW approach with the same dynamical matrix A which is time-independent. Nevertheless the parameters n approaches 0. Thus the ΔV can be computed with the following equation :

$$\overrightarrow{\Delta V} = \frac{\overrightarrow{r_f} - \overrightarrow{r_i}}{\Delta t} - \overrightarrow{v_i} \tag{11}$$

$\overrightarrow{r_i}$: initial position vector
$\overrightarrow{r_f}$: final position vector
Δt : transfer time
$\overrightarrow{v_i}$: initial velocity vector

These algorithms are well-adapted to keplerian orbits. So frames shift toward a Lunar-centered inertial frame (LCI), an Earth-Moon Barycenter-centered inertial frame (BCI) and an Earth-centered inertial frame (ECI) have been applied to DRO. But none of them makes DRO looks like a keplerian orbit (see Figure 7).

The last algorithm, Linearized Relative Targeting (LR) uses more complex formula developed by Luquette [9], and A is now time-dependent as it can be seen in the following equations :

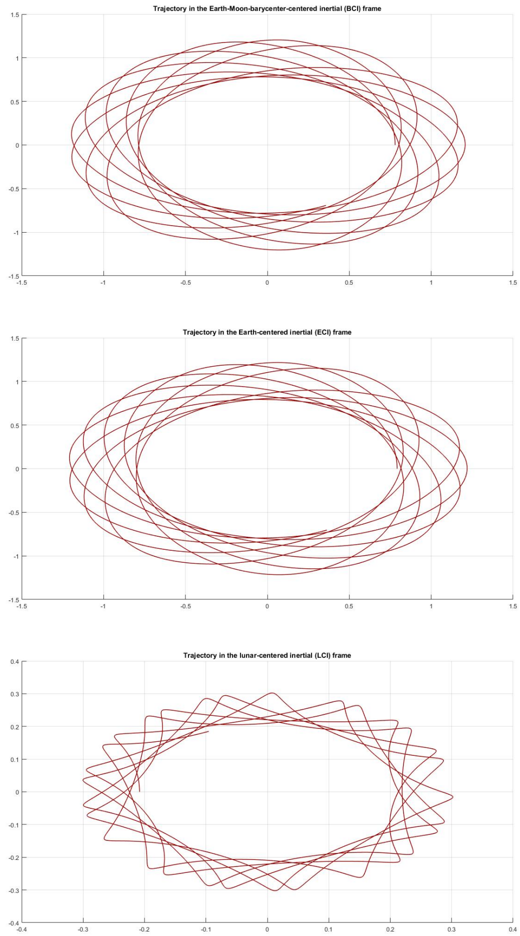

Figure 7: 15 periods of a DRO of Ax = $80 \cdot 10^3$km in BCI, ECI and LCI

$$\dot{X} = \begin{bmatrix} 0 & I_3 \\ \Theta(t) - [n]^2 & -2[n] \end{bmatrix} X \tag{12}$$

With :

$$[n] = n \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{13}$$

(n is the angular rate of change of the LVLH frame)

$$\begin{aligned} \Theta(t) = &-\left(\frac{\mu_1}{||\overrightarrow{r_{1L}}||^3} + \frac{\mu_2}{||\overrightarrow{r_{2L}}||^3}\right) I_3 + \frac{3\mu_1}{||\overrightarrow{r_{1L}}||^3}[\overrightarrow{e_{1L}}.\overrightarrow{e_{1L}}^t] \\ &+ \frac{3\mu_2}{||\overrightarrow{r_{2L}}||^3}[\overrightarrow{e_{2L}}.\overrightarrow{e_{2L}}^t] \end{aligned} \tag{14}$$

Since A is time-dependent, a solution consists in dividing each transfer time into a set of time intervals where the matrix can be considered as constant. Despite its higher complexity, the LR targeting is far more accurate than the other algorithms, and its validity domain is also wider. In fact only CW gives the best results when the rendezvous occurs on a DRO with amplitude under $50 \cdot 10^3$ km. However the size of the orbits is not the only criteria that matters in comparing those algorithms. The position of the rendezvous on the DRO is as important. The angle reference is chosen on the Earth-Moon axis in the synodic frame. The angle grows clockwise. In fact it is the same definition as the one used by Murakami [5].

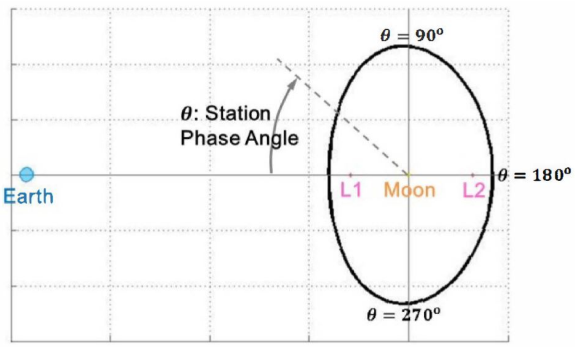

Figure 8: Definition of the station-phase angle from [5]

The symmetry of DRO along the Earth-Moon axis can be observed in Figures 9 and 10 which are symmetric around 180 degree. When the amplitude of a DRO, A_x is below $40 \cdot 10^3$ km in the synodic frame, the DRO is almost circular and thus CW is the best. Between $50 \cdot 10^3$ km and $60 \cdot 10^3$ km, LR reaches the precision of CW. For greater values of A_x, LR is more precise and have less variations, thus the rendezvous can be implemented in a wider zone. CW and SL do not allow such large zone, they are only accurate at very specific angles. When A_x is between 50 and $100 \cdot 10^3$ km the best angle for the rendezvous is around 88 degree using LR.

3.4 Preliminary Trajectory Design

Because of safety procedures a chaser is not permitted to approach the target freely. On the trajectory, some hold points are defined and the chaser must follow a predetermined path that links each of those hold points. A direct approach is never allowed, because in case of a thruster failure, braking may become impossible and jeopardize the target. Three preliminary trajectory methods are proposed by Mand [8], but only two of them are implemented . The other one, called the Double co-Elliptical trajectory has been used by NASA on Low-Earth orbits (LEO) and is more adapted on this kind of orbits. Indeed some maneuvers require too much time in cislunar space. For instance to reduce the Downrange distance between the chaser and the target on LEO, a small gap between the two trajectories is enough for the natural motion to bring them closer (Figure 11). Indeed, on a circular orbit, the higher the spacecraft is, the lower his velocity is. Thus there is an angular speed difference that enables the chaser to catch with the target. The equation 15 gives the catching speed $\dot{x}_0$:

$$\dot{x}_0 = \frac{3}{2}\omega z_0 \tag{15}$$

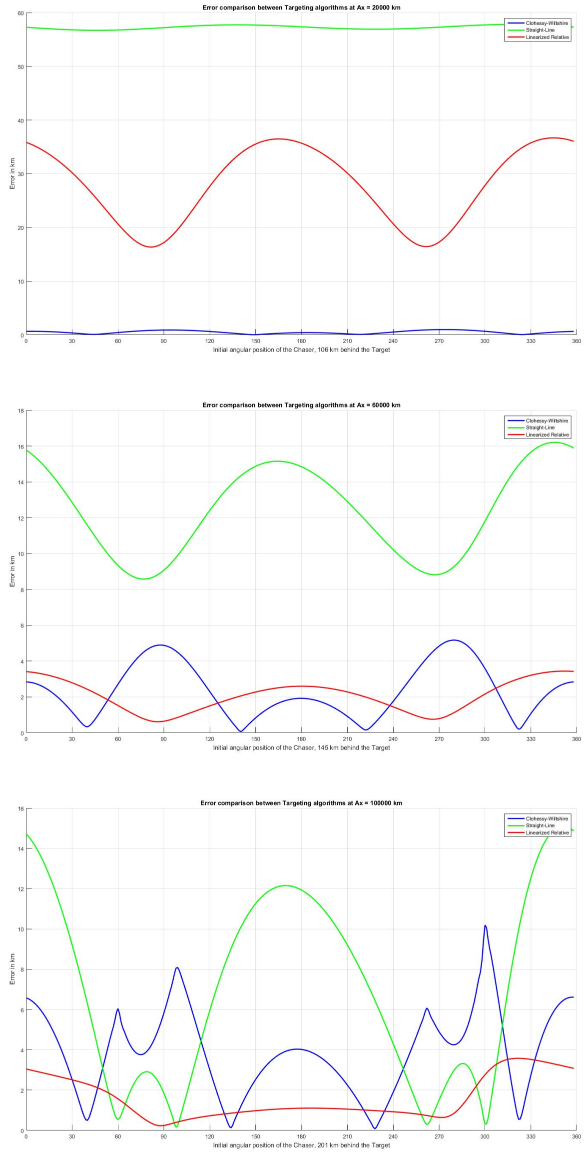

Figure 9: Targeting algorithm comparison with $A_x = 20 \cdot 10^3$ km, $60 \cdot 10^3$ km, $100 \cdot 10^3$ km

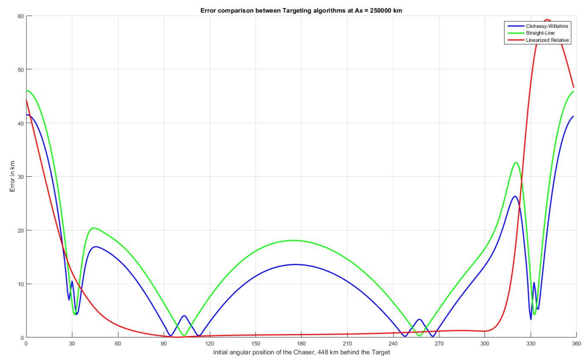

Figure 10: Targeting algorithm comparison with $A_x = 250 \cdot 10^3$ km

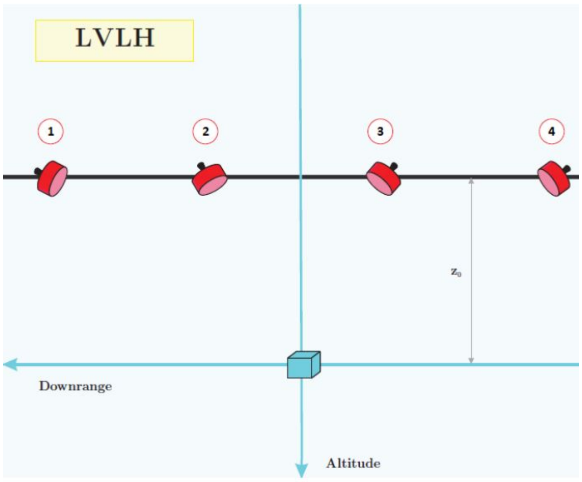

Figure 11: Chaser catching the Target in LVLH frame from [8]

where ω is the angular rate of change of the orbit, and z_0 can be considered constant with typical rendezvous distance : inferior to 100km. The difference between DRO and LEO that matters here, is that ω_{DRO} can be 10^4 times smaller than ω_{LEO}. Therefore 10^4 more time is needed to catch the target with the same initial downrange gap. Such a maneuver time is obviously impossible, that is why this method has not been implemented.

The first preliminary trajectory is the Line-of-Sight Corridor, that defines an approach cone, in which the chaser must stay and do a maneuver every time it reaches a side of the corridor. It is a completely geometrical definition of the path, there is no orbital mechanics behind it.

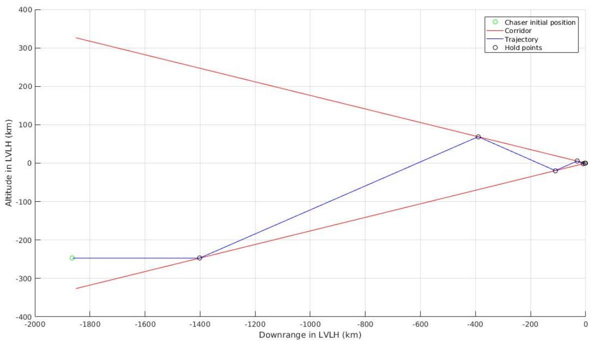

Figure 12: Geometric trajectory of the Corridor rendezvous

An alternative algorithm is the Line-of-Sight Glide which requires more hold points than the corridor, but has a smaller global ΔV in the case studied by Mand [8] and Sara [10].

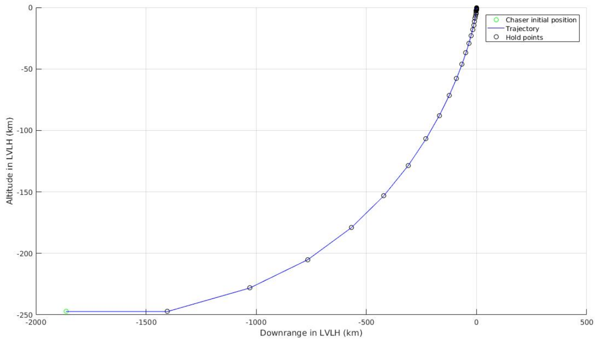

Figure 13: Geometric trajectory of the Glide rendezvous

Those methods are based on the only modifications of the angles of the trajectory, and thus it is easier to make the rendezvous autonomous. During the operations, the Target is always visible by the Chaser, which is a great advantage compared to other methods.

These approaches have been compared in the case of a rendezvous on a DRO. Numerous parameters have a great influence over the results, five were studied :

A_x : amplitude of the DRO
θ : initial angular position
α : upper corridor angle
β : lower corridor angle
ϕ : offset angle

The last three angles are defined on Figures 14 and 15 .

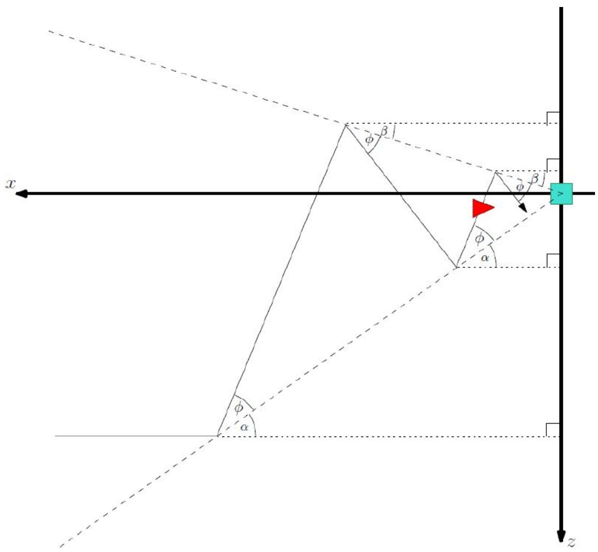

Figure 14: Definition of the angles in the Corridor method (from [8])

The study has been done with these parameters:

- the chaser starts 100km behind the target

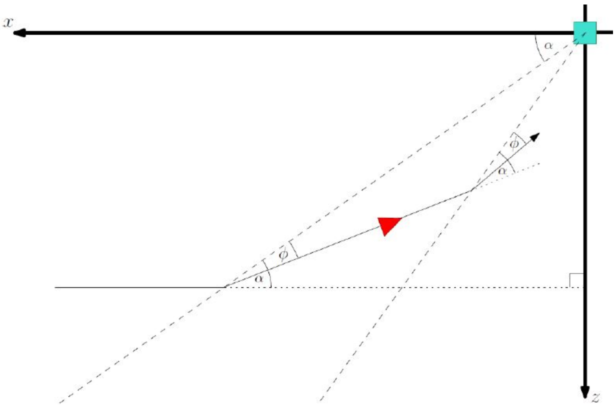

Figure 15: Definition of the angles in the Glide method (from [8])

- Time Of Flight (TOF) : 8 hours

It appears clearly that the Corridor is better than the Glide method. The latter is highly dependent on θ and several peaks can be seen 16: one around 55 degrees and the other around 245 degrees. The curve is not smooth

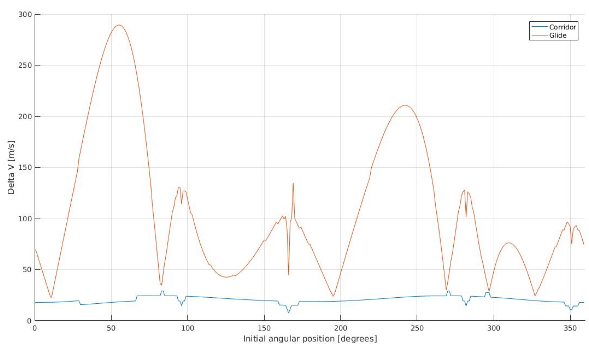

Figure 16: ΔV for the Corridor and Glide methods, Ax = 70x10^{3}km, $\alpha = \beta = 10$ degrees, $\phi = 5$ degrees

at all and present several irregularities. On the whole the graph is always the same, with the Corridor ΔV curve around 20 m/s, and the Glide one well above at some angles. When the size of the DRO increases, the ΔV required for the Glide method increases by 100m/s for 10x10^{3}km. The Glide method gets better when the offset angle is reduced from 5 to 3 degrees. But the best improvement is when the corridor angles increases from 10 to 15 degrees. Under these conditions, the peak of the Glide method is under 75m/s. On the contrary the ΔV of the Corridor is steady around 20m/s.

To sum up, the best way to compute a rendezvous on a DRO, is to use the Line-of-Sight Corridor to get the hold points, and use LR for the calculations. The size of the orbit is a constraint, but the angular position of the rendezvous can be considered as a parameter to reduce the cost of the mission. Since θ does not affect significantly the ΔV, it is better to choose an angle that ensure a good precision with LR, so around 90 degrees.

4 Optimizations

4.1 A limit of the CR3BP

The CR3BP is an idealized model for the Three-Body problem with the assumption of a circular motion for the two primaries. However the relative motion of the Earth and the Moon is not a perfect circle. Therefore slight differences occurs when ignoring this hypothesis. To assess the differences, General Mission Analysis Tool (GMAT) has been used to compute DRO.

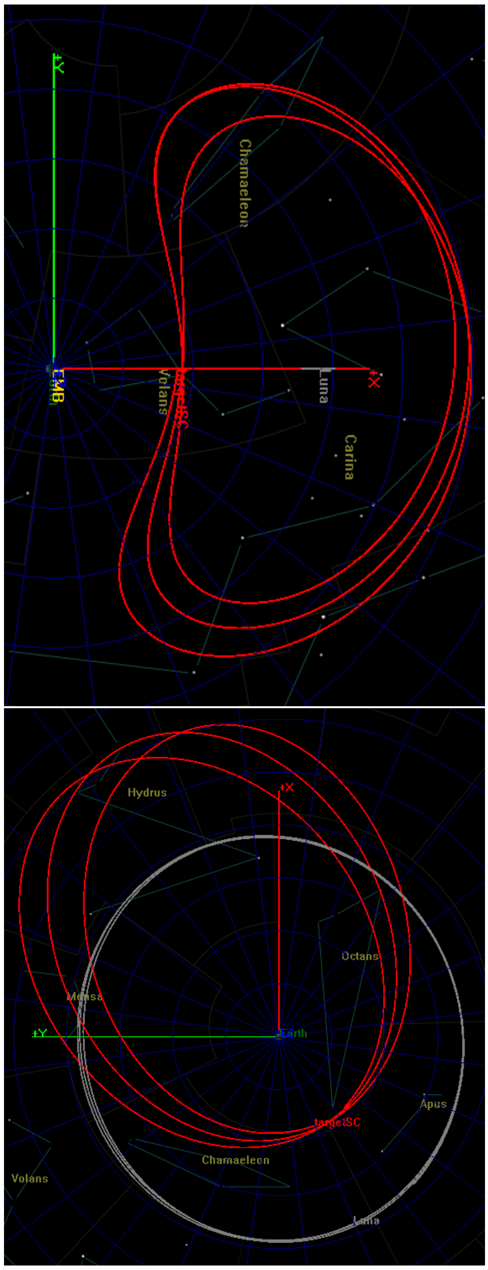

Figure 17: A DRO of amplitude Ax=190,000 km over 3 periods in the synodic frame and in an Earth-centered inertial frame (the axis are not the same)

DRO have been computed with the integrated differential correction solver in GMAT. The initial guesses are identical to those of Murakami and Yamanaka[5] explained in more depth is the equations 1 and 3.

The resulting orbits do differ from those that have been computed with Matlab for the reasons mentioned above,and even a DRO with low amplitude A_x does not appear periodic. After one period, starting with the initial guess, the x component of the position vector is either shifted toward the Earth or the Moon(see Figure 18), which break the periodicity. However, the orbits but do not diverge and remains in the cislunar space. That observation is closely linked with the long-term stability of DRO.

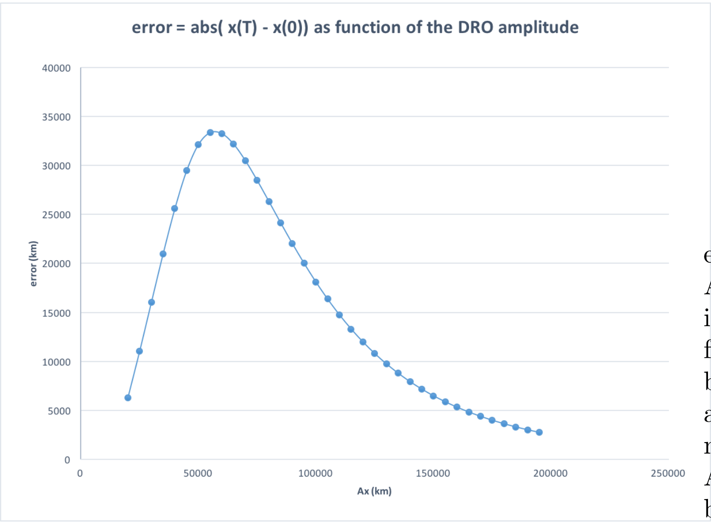

Figure 18: Evolution of the error $\mid x(T) - x(0) \mid$ with respect to A_x over one the period of the DRO

Numerical refinement methods such as multiple shooting [11] has been tried in order to reduce the discrepancy of positions after one period without relevant results. But despite being perturbed, the resulting orbit is comparable with the ones obtained by Murakami (Figure 19) and which were plotted with STK/Astrogator, a software that provides full-ephemeris models of the Sun, Earth and Moon and is used to prepare real space missions similar to GMAT.

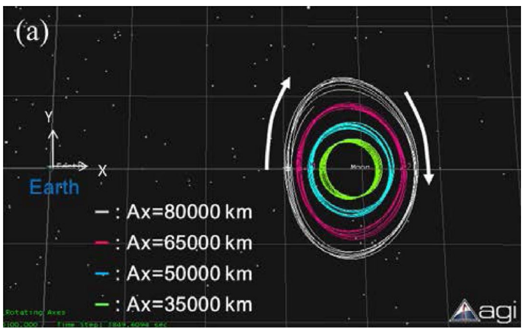

Figure 19: DRO obtained with STK/Astrogator in [5]

4.2 Error propagation on a Rendezvous trajectory

Safety is a crucial point in a space mission, and even trajectory design has to consider it. The errors can come from three fields : the construction of the trajectory; the navigation, which ensure the chaser stays on his trajectory; the error of position due to the sensors. Approach procedures to the ISS have been designed with several step. The Approach Ellipsoid (AE) is an

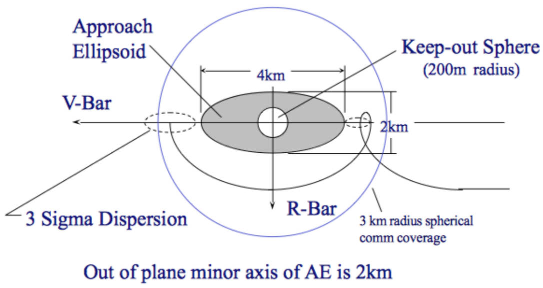

Figure 20: ISS approach ellipsoid

ellipse of 4 x 2 x 2km dimensions, surrounding the ISS. A Visiting Vehicle (VV) can only be allowed to enter it under specific conditions to ensure there are no risks for the ISS. It must receive authorizations at each step before proceeding to the next one. The approach zone is an ellipse since on LEO there is a preferential direction of motion : along the V-bar axis, since orbits are circular. And thus the AE is bigger in this special direction because the speed of a VV is also greater along this axis. However in deep space there is no such direction, VV can come from every direction. Therefore, an Approach Sphere seems a better idea than the Ellipsoid.

The Keep Out Sphere (KOS) is a 200m radius sphere centered around the ISS center-of-mass. If an error occurs during an approach maneuver the free drift of the Chaser must not enter the KOS. The free drift is the state of motion when no machine is used to control it. The vehicle is only subjected to its inertia and to the local gravitation potential. In order to model the errors, a white Gaussian noise has been added on the direction and the norm of the ΔV at each maneuver that occurs at every hold point. A 24 hours propagation of every perturbed trajectory has been done to ensure that none of them entered the KOS.

4.3 Code optimization

There are loads of safety calculations to be made and thus the algorithms must be optimized. Several steps can be implemented. The first one is by the gathering of the three targeting methods : SL, CW and LR in one more global method.

The function that takes the most time is the propagation of the state vector and state transition matrix along the trajectory, which is the trajectory calculation in fact. Since there are several hold points during a rendezvous, this propagation is computed for each trajectory between those points. Parallel computing is the solution to this problem. Yet the gain is limited by the number of core of a computer, and some time is required to initialize and control each processor. That is why a great amount of calculations is needed for the parallelization to be efficient. A single propagation is not enough, it is worth the investment when used on error propagation on a broad range of orbits and angular positions of DRO.

Another solution is the use of a faster language, for instance C++ instead of Matlab, which is very practical, but too slow. Thus the propagation is computed in C++ and then a MEX file (Matlab Executable) links it to the rest of the code written in Matlab. The propagation of the equation is accelerated by a 6.4 factor, and the whole rendezvous calculation is in average 2.3 times faster when using MEX files.

5 Future work

As shown in this paper, existing methods to compute the ΔV for a rendezvous in a DRO are efficients within different domains. A possible continuation of this research is to find an algorithm that combines these three methods together to aim at efficiency and accuracy. Another step of this research could be focused on the compromise between ΔV and the time of flight in term of DRO amplitudes and the places to be selected for a rendezvous. Finally an extension of this research could explore in more depth safety analysis and trajectory dispersion in case of thruster failure in the vicinity of the target. The preliminary study done in [10] could be a good start.

6 Conclusion

The initial goal of our research was to determine the most adapted methods to make a rendezvous on a DRO among Clohessy-Wiltshire (CW), Linearized Rectilinear (LR) and Straight Line (SL) approaches for the relative equations of motion. Among those methods, the SL approach is ruled out since it is often less efficient that CW and LR. Compared to LR, CW gives the best results when the target's orbit is of low amplitude (below $50 \cdot 10^3 km$) and can be approximated by a moon-centered circular orbit. For other DRO with an amplitude that is higher than $50 \cdot 10^3 km$, and except from some positions for the rendezvous on a DRO, the LR approach gives the best results.

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